

Quiz 5

September 14, 2016

Show all work and circle your final answer.

1. Evaluate $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^4 - 27x^3 + 3}}{x^2 + x + 1} \cdot \frac{1/\sqrt{x^4}}{1/x^2}$ ($x^2 = +\sqrt{x^4}$ for all values of x)

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{1 - 27/x + 3/x^4}}{1 + 1/x + 1/x^2} = \frac{\sqrt{1}}{1} = \boxed{1}$$

2. Evaluate the following limits:

(a) $\lim_{x \rightarrow \infty} \sqrt{16x^2 + 5x} = \sqrt{\lim_{x \rightarrow \infty} (16x^2 + 5x)} = \boxed{\infty}$

(b) $\lim_{x \rightarrow \infty} 4x = \boxed{\infty}$

(c) $\lim_{x \rightarrow \infty} (\sqrt{16x^2 + 5x} - 4x) \cdot \left(\frac{\sqrt{16x^2 + 5x} + 4x}{\sqrt{16x^2 + 5x} + 4x} \right)$

$$= \lim_{x \rightarrow \infty} \frac{16x^2 + 5x - 16x^2}{\sqrt{16x^2 + 5x} + 4x} \cdot \frac{1/x}{1/x^2}$$

$$= \lim_{x \rightarrow \infty} \frac{5}{\sqrt{16 + 5/x} + 4} = \boxed{\frac{5}{8}}$$

3. Find the derivative of $f(x) = \sqrt{x+1}$ at $x = a$.

$$f'(a) = \lim_{h \rightarrow 0} \frac{\sqrt{a+h+1} - \sqrt{a+1}}{h} \cdot \frac{\sqrt{a+h+1} + \sqrt{a+1}}{\sqrt{a+h+1} + \sqrt{a+1}}$$

$$= \lim_{h \rightarrow 0} \frac{(a+h+1) - (a+1)}{h(\sqrt{a+h+1} + \sqrt{a+1})} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{a+h+1} + \sqrt{a+1})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{a+h+1} + \sqrt{a+1}} = \boxed{\frac{1}{2\sqrt{a+1}}}$$